Assignments 1 for OIT 102 and OIT 103

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1 General Comments to Assignments 1 for OIT 102 and OIT 103

Generally, you have done very well in your asignments.

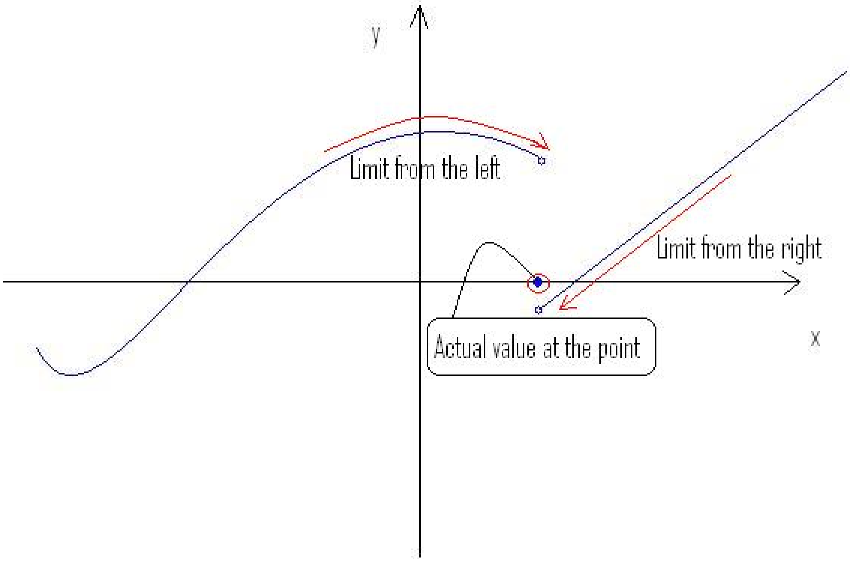
For those of you, including second year students, who have handed in the first assignment for the course OIT 102, you have scored an average of 4.8 points out of 7.5, or an average of 64 %. I received assignment 1 from 32 students.

For those of you, including second year students, who have handed in the first assignment for the course OIT 103, you have scored an average of 5.2 points out of 7.5, or an average of 69 %. I received assignment 1 from 29 students.

There were 2 areas in which most of you had problems, see the next sections. Other than that, the most common error you made was to not read the problem formulation correctly. Make sure that you have read and understood the question before you set out to answer it.

2 Limits

Most of you had problems with the concept of limits. Consider the picture below:



The above function has a discontinuity at the point a. Recall that the limit in a is simply the value (y or $f(x)$) that the function approaches as x approaches a.

If you look at the red arrows, you can see that $lim_{x \to a^-} f(x) \neq lim_{x \to a^+} f(x)$. Most of you had no trouble determining the limits from the left and the right.

The problem arose with the limit $lim_{x \to x} f(x)$. Those of you who did not correctly answer that **this limit doesn't exist**, suggested that the limit is the value $f(a)$. However, consider for a moment if the functional values actually approach $f(a)$ at all. They don't!

Recall again that the limit in a is the value that the function approaches as x approaches a. If this is not clearly defined, meaning that $lim_{x \to a^-} f(x) = lim_{x \to a^+} f(x)$, then we can't assign a value to it.

In the above situation, we have no less than 3 equally qualified candidates for this limit, namely $lim_{x \to a^-} f(x)$, $lim_{x \to a^+} f(x)$ and $f(a)$. As a mathematical quantity can't have 3 different values at the same time, we say that it doesn't exist. It doesn't exist because we can't assign a unique value to it.

3 Mathematical Induction

Very few of you managed to get the questions about mathematical induction right.

First of all, note that in order to carry out a proof by mathematical induction, we need a **hypothesis** (the property we're trying to prove) and **3 steps**:

1) INDUCTION START: This is where you **test the hypothesis** for a small value of n, typically 1 or 2. If you fail to prove your hypothesis for a small value of n, it may be because the hypothesis is false.

2) INDUCTION STEP: After successfully testing the hypothesis, this is where you **assume that the hypothesis holds for all natural numbers** $k \leq n$.

3) INDUCTION CONCLUSION: This is the part where you show that **if the hypothesis holds for n, then it holds for n + 1**.

I will answer the 2 problems from the assignment here below and write comments as I go along:

1) $(ab)^n = a^n b^n$ (Here, it's the exponent n you must use induction on).

INDUCTION START:	$(ab)^1 = ab = a^1 b^1$	Test the hypothesis for $n = 1$
		The first element of $\mathbb{N}$ is 1
	$(ab^2) = abab = aabb = a^2 b^2$	Test the hypothesis for $n = 2$.
		This is not necessary,
		but we do not show much for $n = 1$
INDUCTION STEP:	Assume that the hypothesis is true for $k \leq n$	This is the induction hypothesis
		n is arbitrary but set
INDUCTION CONCLUSION:	$(ab)^{n+1} =$	This is what we want to know something about
	$(ab)^n ab =$	Using the induction hypothesis to get
	$\underbrace{a^n b^n}_{\text{Induction hypothesis}} \quad ab =$	We have assumed that $(ab^n) = a^n b^n$
	$a^{n+1} b^{n+1}$	This is what we wanted to show!

Note that what is important here is to use the INDUCTION HYPOTHESIS to show that the CONCLUSION holds, or in other words we show that **if the hypothesis holds for any n, then it holds for** $n+1$. This is where the nature of the natural numbers come in: We start with one, and if the hypothesis holds for 1, and has the property that every time it holds for n, then it also holds for $n+1$, then we ca conclude that it holds for all $n \in \mathbb{N}$ as we can construct all of them from 1 by adding 1 at a time.

2) $\sum_{i=1}^{n}(n^2 - n) = \frac{n^3-n}{3}$

INDUCTION START:	$1^2 - 1 = 0 = \frac{1^3-1}{3}$	Test the hypothesis for $n = 1$ The first element of $\mathbb{N}$ is 1
	$1^2 - 1 + 2^2 - 2 = 2 = \frac{2^3-2}{3}$	Test the hypothesis for $n = 2$. This is not necessary, but we do not show much for $n = 1$
INDUCTION STEP:	Assume that the hypothesis is true for $k \leq n$	This is the induction hypothesis n is arbitrary but set
INDUCTION CONCLUSION:	$\sum_{i=1}^{n+1} i^2 - i =$	This is what we want to know something about
	$\sum_{i=1}^{n} i^2 - i + (n+1)^2 - (n+1) =$	Using the induction hypothesis to get
	$\underbrace{\frac{n^3 - n}{3}}_{\text{Induction hypothesis}} +(n+1)^3 - (n+1) =$	We have assumed that $\sum_{i=1}^{n} i^2 - i = \frac{n^3-n}{3}$
	$\frac{n^3-n}{3} + \frac{3n^2+3n}{3} =$	
	$\frac{n^3+3n^2+2n}{3} =$	
	$\frac{(n+1)^3-(n+1)}{3}$	This is what we wanted to show!

You're strongly encouraged to carry out the calculations yourself.

Note that again, the method is that of using the assumption to prove that the desired property also holds for the next natural number. As we have shown it for some natural number n, but we haven't attached a value to n, we may conclude that when it holds for 1 or 2, and it holds for $n+1$ whenever it holds for n, then it holds for all natural numbers.