Inverting Matrices

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Inverting a 2x2 Matrix

To invert a non-singular 2x2 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

you calculate the determinant $det(A)$, and the inverse is given by

$$A^{-1} = \frac{1}{det(A)} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Inverting a 3x3 Matrix

Given a non-singular 3x3 matrix

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

You calculate the determinant $det(B)$, and then you replace each entry of B in the following way:

Delete the row and column that b_{ij} is an entry of. The result is a 2x2 matrix, $\hat{B_{ij}}$. Calculate the determinant of that matrix and replace b_{ij} with $(-1)^{i+j} det(\hat{B_{ij}})$.

E.g. b_{11} is replaced by

$$det(\begin{bmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{bmatrix})$$

and b_{12} is replaced by

$$-det(\begin{bmatrix} b_{21} & b_{23} \\ b_{31} & b_{33} \end{bmatrix})$$

Eventually you transpose the resulting matrix, and all in all you get:

$$B^{-1} = \frac{1}{det(B)} \begin{bmatrix} b_{22}b_{33} - b_{32}b_{23} & -(b_{12}b_{33} - b_{32}b_{13}) & b_{12}b_{23} - b_{22}b_{13} \\ -(b_{21}b_{23} - b_{31}b_{23}) & b_{33}b_{11} - b_{31}b_{13} & -(b_{23}b_{11} - b_{21}b_{13}) \\ b_{21}b_{32} - b_{31}b_{23} & -(b_{11}b_{32} - b_{31}b_{12}) & b_{11}b_{22} - b_{21}b_{12} \end{bmatrix}$$