

# MODULE 1

## ***THE REAL NUMBER SYSTEM***

### ***(Natural, Integers, Rational and Irrational Numbers)***

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#### **1.1 Introduction**

Mathematics begins with arithmetic, the art of manipulating numbers using the arithmetic operations of addition, subtraction, multiplication and division.

#### **1.2 Objectives**

At the end of this module the learner will be able to:

- Differentiate between natural numbers, integers, rational and irrational numbers.
- Appreciate the need for the systematic development of the real number system.
- Distinguish between discrete and continuous variables.

### 1.3 Natural numbers

Natural numbers are synonymous to counting numbers. We denote the set of natural numbers by the letter  $N$ . The set begins with the number 1 (one) and continues to numbers bigger than 1 by adding one to a previous number to obtain the next number. The **sum** or **product** of natural numbers is again a natural number. However, **subtraction** of natural numbers does not always result in a natural number. Typically,  $5 - 7$  is not a natural number.

### 1.4 Integer Numbers

To overcome the problem of division we expand the set of natural numbers to include the number zero and the negatives of the natural numbers. The enlarged set is called the set of integers, denoted simply by the letter  $Z$ . The set of natural numbers is a proper subset of the set of integers.

Within the set of integers, addition, subtraction and multiplication result in integers. However, **division** (by a nonzero integer) does not always result in an integer. Typically,  $\frac{3}{4}$  is not an integer.

### 1.5 Rational numbers

To allow for unrestricted division we expand the set of integers. The new set is called the set of rational numbers, which we denote by the letter  $Q$ . A rational number has the general form  $\frac{p}{q}$ , where  $p$  and  $q$  are integers numbers with  $q \neq 0$ . The set of integers is a proper subset of the set of rational numbers.

It is tempting to think that the set  $Q$  of rational numbers contains all the numbers we would ever wish to operate with. This is not the case. The real numbers have a one-to-one correspondence with points on the number line. However, there are some real numbers on the number line which are not rational numbers. This means that some real numbers cannot be expressed in the form  $p/q$ . A typical example is  $\sqrt{2}$ .

## 1.6 Irrational Numbers

In order to capture all real numbers we must therefore introduce a new set of numbers, those that are real but not rational. These are called irrational numbers. Let us denote the new set by  $IR$ .

Clearly, the sets  $Q$  and  $IR$  are mutually disjoint, they have no element in common, their intersection is the empty set and their union is the set of real numbers  $\mathfrak{R}$ .

## 1.7 Summary

The real number system is the union of the set of rational numbers and the set of irrational numbers. The rational numbers includes all the natural numbers and the integers. Numbers are discrete variables, and have been introduced in this first module to prepare the learner for understanding the continuous variables that are inextricably linked to the all important concept of functions and the associated concepts of limits, continuity and differentiation.