Module 1: Basic Set Concepts

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Contents

1 Introduction:

Sets and set operations appear in a large number of mathematical concepts and methods as well as forming a basis for many operations that we do not normally connect with sets.

This module corresponds to sections 1.1, 1.2, 1.3, 1.4 and 1.5 in the chapter on sets.

2 Objectives:

At the end of this module the learner will be able to:

- Identify a set and the most common ways of writing sets

- Distinguish whether an item is an element of a given set or not

- Perform basic set operations such as union and intersection

- Use Venn diagrams to explain set theoretic concepts

- Tell the difference between subset and superset

3 Sets and the most common ways of writing them:

A set is a collection of items. The set is made up by these items. The items that make up a set determine it completely. Usually, but not always, the items that form the set will have some property in common. Sets can be finite (meaning that they consist of a finite number of items) or infinite (meaning that they consist of so many items that they can never be counted).

Common ways of writing a set is

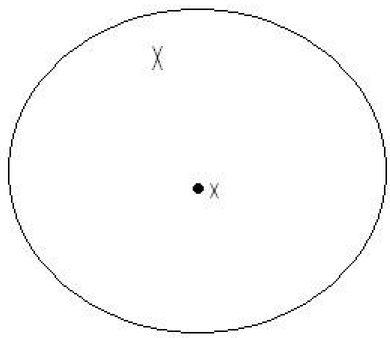
$\{1, 2, 3, 4, 5\}, A, B, C, \{2, 4, 6, 8, \ldots\}, \{\ldots, -2, -1, 0, 1, 2, \ldots\}, \ldots$

Some very important sets are the universal set, U, which in a sense describes the universe in which we work, "the set of everything", and $\emptyset$, the empty set. This means that $\emptyset$ is the set that is defined by having no items in it. Though defining these 2 sets may seem superfluous, they do actually play important roles in mathematics.

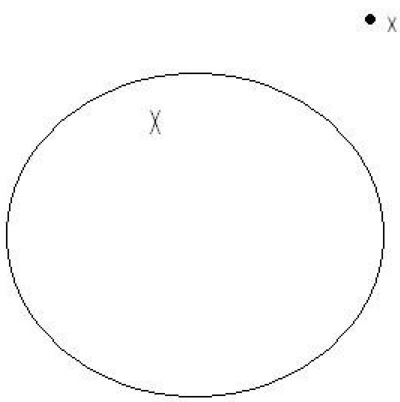
4 Elements of sets:

The items that a set is made up of are called elements. We speak of the elements of a set, and say that a set is completely determined by its elements. If a set is large, it is more feasible to describe it by some common property of its elements than by listing all the elements of it.

To say that some entity, say x, is an element of a set, say X, we write $x \in X$:



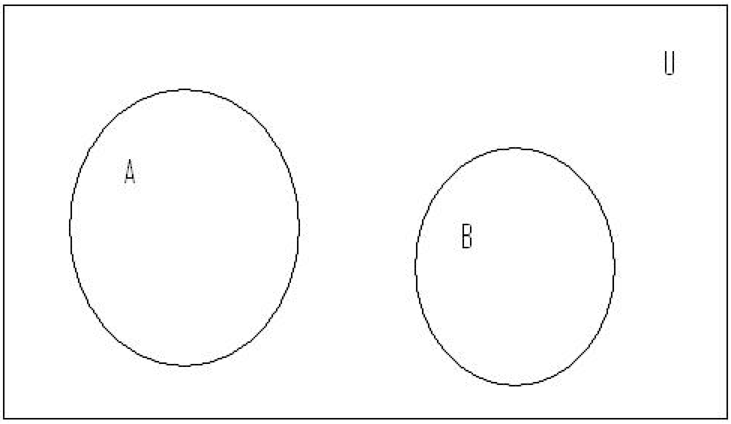
To say the opposite, that x is not an element of X, we write $x \notin X$:



A way of listing A by its elements is $A = \{x : x \text{ has a certain property}\}$ or $A = \{x|x \text{ has a certain property}\}$. The colon or vertical line is read as "which" or "such that", so A is the set of x which has a certain property, or the set of x such that x has a certain property.

5 Venn diagrams:

A very graphically appealing way of understanding sets is by the so-called Venn diagrams. We make a large square box to illustrate U, the universal set, and make circles to illustrate the sets that weŠre interested in, say A and B:



This is to be interpreted as follows: The circle labelled "A" contains all the elements that the set A is determined by, and that the circle labelled "B" contains all the elements that the set B is determined by. In this case, A and B have no elements in common.

6 Union and intersection:

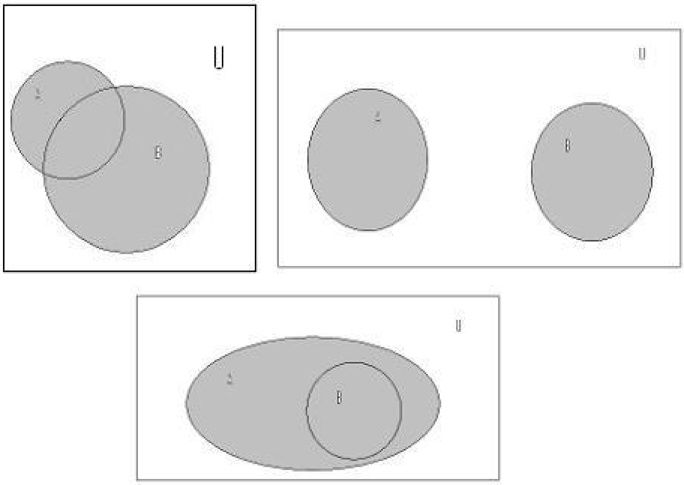
2 very common and very important set operations are the union of 2 sets and the intersection of 2 sets.

Using Venn diagrams to illustrate the concepts, we have that the union of 2 sets A and B are:

all the elements of A, together with

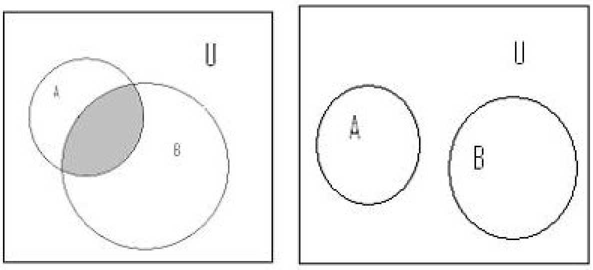
all the elements of B, together with

all the elements that are elements of A and B at the same time:



The above Venn diagrams all illustrate the union of 2 sets A and B. We write the union of 2 sets A and B as $A \cup B$.

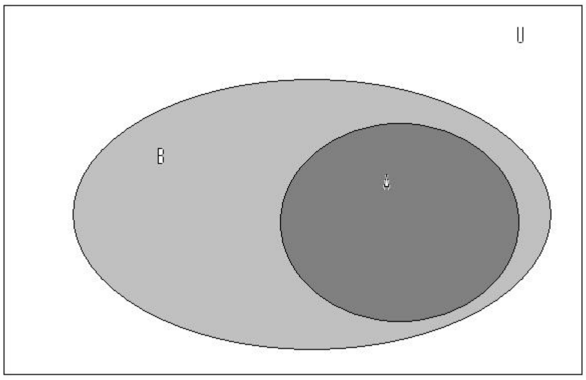
Similarly, the intersection of A and B, written $A \cap B$, denotes all the elements that are elements of A and B at the same time:



In case there are no elements that are elements of both A and B at the same time, we say that the intersection is empty, or that it equals the empty set, $\emptyset$. In symbols: $A \cap B = \emptyset$.

7 Subset and Superset

If all the elements in a set A are also elements of a set B, we say that A is a subset of B:



We write this $A \subset B$ or $A \subseteq B$. In this case, we say that B is a superset of A, which we write $B \supset A$ or $B \supseteq A$. The term superset is not commonly used, though.