Module 5: Introduction to (Formal) Logic

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1 Introduction:

Logic is the basis of how we mathematicians argue. In order to be sure that an argument or a proof holds, we must follow the rules of (formal) logic in every step of the justification of it. The mathematical logic, or (formal) logic, doesn't look very similar to what we call logic in our everyday lives, but as we get more comfortable with it, we'll soon recognise that we use the same logical structures to justify arguments in spoken English - or kiSwahili.

This module corresponds to sections 1.1 and 1.2 in the chapter on logic.

2 Objectives:

At the end of this module the learner will be able to:

- Distinguish between (logical) and, and (logical) or

- Understand the use of (logical) not

- Work with truth tables

3 Propositions

A proposition as a statement that can be either true or false. If no truth value can be determined for some sentence, then it is not a proposition. Here are some examples of propositions:

"My math teacher is a lady."

"Some bananas can be cooked while they are green."

"People can drink water."

"All cats in Tanzania are grey."

"This sentence is written in kiSwahili."

The frist proposition is true if the math teacher is a lady, and false if the math teacher is a man. The second and third propositions are obviously true as we cook green bananas and drink water, while the fourth proposition will be proven wrong by the first cat we see that isn't grey. The last proposition is obviously false as it is written in English.

Here are some examples of sentences which are not propositions. Try assigning truth values for them yourselves

"It will snow in Dar es Salaam 3 months from today."

"x is an integer."

"If this sentence is true, then this sentence is false, but if it is false, then it is true."

Note that the first sentence is verifiable: If it snows in Dar 3 months from today, then it is true, but if it doesn't snow in Dar 3 months from today, then it is false. However, we can't know what will happen 3 months from today, however unlikely it is that it will ever snow in Dar es Salaam.

Note that propositions are sometimes called statements or claims.

4 Logical And

The logic "and" and the logic "or" differs somewhat (but not a lot) from the way we use "and" and "or" in our everyday conversations. Let's start with the logic "and":

Given 2 propositions **proposition 1** and **proposition 2**, we say that the composite proposition **proposition 1** ***and*** **proposition 2** is true if both of them are true. To look at our examples from above, if we take 2 true propositions:

"Some bananas can be cooked while they are green."

"People can drink water."

We can make the composite proposition:

"Some bananas can be cooked while they are green **and** people can drink water."

which is obviously still true. However, if we make a composite proposition in which one of the original propositions is false, then the composite proposition is false. Hence both of the following propositions are false:

"People can drink water **and** all cats in Tanzania are grey."

"This sentence is written in kiSwahili **and** some bananas can be cooked while they are green."

If our 2 propositions are called p and q, then we write "p and q" as $p \wedge q$.

Note that you can make composite propositions with more than just 2 propositions.

5 Logical Or

We can make composite propositions using logical or just as we made them using logical and. The difference is that with a logical or, it is enough for one of the original propositions to be true. The composite proposition will also be true if both of them are true. It will only be false if both of them are false. The following 3 composite propositions are true:

"Some bananas can be cooked while they are green **or** people can drink water."

"People can drink water **or** all cats in Tanzania are grey."

"This sentence is written in kiSwahili **or** some bananas can be cooked while they are green."

The following composite proposition is false:

"All cats in Tanzania are grey **or** this sentence is written in kiSwahili."

If our 2 propositions are called p and q, then we write "p or q" as $p \vee q$.

6 Logical Not

Just as we can negate the meaning of a sentence in English, we can negate the truth value of a proposition. Consider the following:

"All cats in Tanzania are grey."

"All cats in Tanzania are **not** grey."

"Some bananas can be cooked while they are green."

"**No** bananas can be cooked while they are green."

Note that the way we negate a spoken language sentence differs a bit with the structure of the sentence. To negate a proposition "p", we write "¬ p". If you negate a true proposition, it becomes false, and if you negate a false proposition, it becomes true.

7 Truth Tables

A truth table is a table that displays the truth value of a proposition. If we have only one proposition, say p, then the truth table for p and $\neg p$ will look like this:

p	$\neg p$
True	False
False	True

If we try adding another proposition, say q and look at $p \wedge q$ (p and q) and $p \vee q$ (p or q), the truth table becomes more complicated:

p	q	$p \wedge q$	$p \vee q$
True	True	True	True
True	False	False	True
False	True	False	True
False	False	False	False