## REVIEW QUESTIONS

### Module 2: Functions

1. What is the difference between a discrete and a continuous variable? Give suitable illustrate examples

2. Define precisely the concept of a real valued function $f(x)$ whose domain is $D$ and range is $R$.

3. Give a mathematical definition of the limit concept for a function $f(x)$ as $x$ approaches an interior point $c$ in its domain.

4. Given the functions (i) $f(x)=\begin{cases} x^2+1 & if \quad x>1 \\ 3-2x & if \quad x\le 1 \end{cases}$

   (ii) $g(x)=\begin{cases} \dfrac{x^3-1}{x-1} & if \quad x\ne 1 \\ 1 & if \quad x=1 \end{cases}$, find the following limits if they exist:

   $\lim\limits_{x\to 1^-} f(x)$; $\lim\limits_{x\to 1^+} f(x)$; $\lim\limits_{x\to 1} f(x)$; $\lim\limits_{x\to 1^{-1}} g(x)$; $\lim\limits_{x\to 1^{+1}} g(x)$; $\lim\limits_{x\to 1} g(x)$

5. Evaluate the following limits

   (i) $\lim\limits_{x\to 1^+}\left[\dfrac{x^3-1}{x-1}\right]$.

   (ii) $\lim\limits_{x\to 1^-}\left[\dfrac{x^3-1}{|x-1|}\right]$.

   (iii) $\lim\limits_{x\to\infty}\left[\dfrac{1}{x^2}\right]$.

   (iv) $\lim\limits_{x\to\infty}\left[\dfrac{x^2+1}{3x^3+10}\right]$

   (v) $\lim\limits_{x\to\infty}\left[\dfrac{x^3-2}{3x^3+2x-3}\right]$.

6. Define the concept of continuity of a function $f(x)$ at a point $x=c$. Use the definition to determine values of the parameters $a$ and $b$ such that the function

$$f(x) = \begin{cases} \dfrac{a}{x-2} & for \quad x \le 0 \\ 2x+b & for \quad 0 < x < 2 \\ 6 & for \quad x \ge 2 \end{cases} \qquad \text{is continuous everywhere.}$$

7. Using the concept of limits give the mathematical definition of the derivative of a function at a point $x = c$ and use the definition to determine the derivative of the function $f(x) = \sqrt{x}$

8. Using the method of substitution, find the anti-derivative of each of the following functions:

(a) $f(x) = \dfrac{1}{x}\ln(x)$

(b) $f(x) = \dfrac{e^x}{\sqrt{1-e^x}}$

(c) $f(x) = \dfrac{\sin(x)}{\cos(x)}$

9. Apply the method of integration by parts in finding anti-derivatives and hence evaluate the integrals

(a) $\int_0^1 xe^x dx$

(b) $\int_1^2 \ln(x)dx$

(c) $\int_0^{\frac{\pi}{2}} x\cos(x)dx$

10. Let $I_n = \int_0^{\pi} \sin^n(x)dx$.

(a) Find $I_0$.

(b) By expressing the integrand in the form $\sin^n(x) = \sin(x)\sin^{n-1}(x)$ and integrating by parts, show that $I_n = \dfrac{n-1}{n} I_{n-2}$.

(c) Use the above results to evaluate $\int_0^{\pi} \sin^2(x)dx$.