REVIEW QUESTIONS

Module 6: Numerical Integration

1. Without resorting to any numerical method, evaluate if you can, the definite integral $\int_0^1 f(x)dx$, where $f(x)$ is the function tabulated below.

x	0.0	0.125	0.25	0.375	0.5	0.675	0.75	0.875	1.0
$f(x)$	1.0	2.51	3.23	5.62	7.37	10.55	9.63	8.26	5.33

If you are unable to solve the problem explain clearly why and use this problem to give a number of sound mathematical reasons why numerical integration methods are very much needed.

2. Write down the general format of a numerical integration method and use it to point out the essential characteristics of the Newton-Cotes family of numerical integration methods. Mention two numerical integration methods belonging to this family of methods.

3. (a) Derive the formula for the Trapezium rule over a single interval $[x_0, \quad x_1]$.
   (b) Show that the Trapezium rule is exact for all linear function $f(x) = ax + b$. Note that $x_1 = x_0 + h$, and that for simplicity of manipulations, you may take $x_0 = 0$.
   (c) se it to obtain the extended Trapezium rule for the general integral $\int_a^b f(x)dx$.

4. Write down the formula for Simpson's rule over the double interval $[x_0, \quad x_1, \quad x_2]$.
   Show that Simpson's rule is exact for all quadratic functions $f(x) = ax^2 + bx + c$. Note that $x_1 = x_0 + h, \quad x_2 = x_0 + 2h$, and that for simplicity of manipulations, you may take $x_0 = 0$.

5. (a) Evaluate analytically the definite integral $\int_0^1 \frac{x}{1+x^2} dx$, rounding your answer correct to six decimal places accuracy.

(b) Construct a table of values of the function (integrand) $f(x) = \frac{x}{1+x^2}$ over the range of integration $[0,\ 1]$ at equidistant points using an interval of length 0.125 .

(c) With the aid of the table of values you have constructed approximate the definite integral $\int_0^1 \frac{x}{1+x^2}\,dx$ using:

(i) The Trapezium rule with $h = 0.25$
(ii) The Trapezium Rule with $h = 0.125$
(iii) Simpson's rule with $h = 0.25$
(iv) Simpson's rule with $h = 0.125$

6. From the numerical results you have calculated and by comparing the results with the exact value obtained in part (a) of this question, make comments on

(a) The accuracy of the Trapezium rule compared with Simpson's rule for the same interval length h.

(b) The accuracy of the Trapezium rule as the interval length h gets smaller.

(c) The accuracy of Simpson's rule as the interval length h gets smaller.