VISTOOMA, VIsualisation TOOl for MAth. Module 1: Venn Diagrams and Set Algebra

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1 Module 1: Venn Diagrams and Set Algebra

Students normally don't have any problems with understanding the concept of Venn diagrams, though they might not always draw them correctly. The challenges in basic set theory is understanding the set algebraic and "mathematical" ways of describing sets and manipulating these expressions.

This module aims at facilitating the user in understanding the basics of

set algebra, simultaneously using Venn diagrams, set algebraic notation and mathematical notation to assist in making the connection between the 3 ways of describing sets.

1.1 Software Requirements for Module 1: Venn Diagrams and Set Algebra

These are the functionalities that are needed for Vistooma to have the desired educational potential.

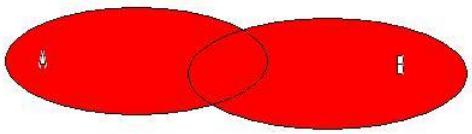
1.1.1 Software Requirements for Constructin Sets (The basics)

Vistooma is an interactive graphical user interface. It needs to be programmed in a programming language that facilitates keeping track of movable, renameable and resizeable geometrical objects (the sets) no matter where you reposition them on the screen.

There are basically 3 ways of describing a set: As a Venn diagram (a drawing), using a set algebraic description (such as $A \cap B$, or using a "mathematical" description (such as $\{x|x \in A \wedge x \in B\}$). Students seem to have some trouble especially with the last way of describing sets, which is why Vistooma simultaneously gives all 3 descriptions to allow them to see the connection.

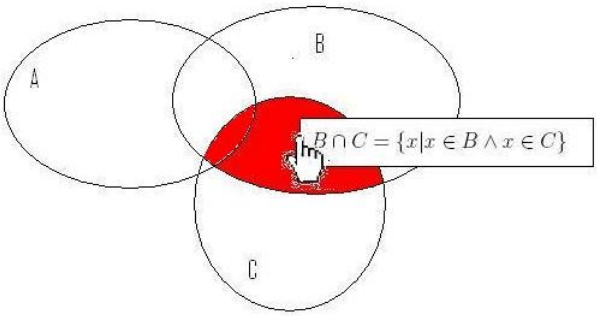
Vistooma needs to have the ability to identify all 3 ways of describing sets with each other, and to generate the other 2 types of description from the third no matter which type of description is given. The graphical user interface part will allow for the geometrical expression of the sets, but it needs to have a dialogue box in which to type in the other 2 descriptions as well.

That is, if you give Vistooma either of the descriptions "$A \cup B$" and "$\{x|x \in A \vee x \in B\}$", it will generate the following picture, which could also have been created by generating 2 sets and dragging them around physically to position them with a bit overlapping each other:



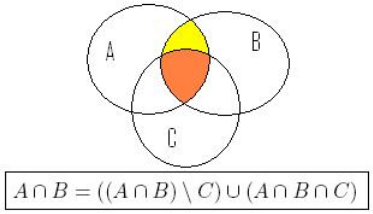
$A \cup B = \{x|x \in A \vee x \in B\}$

Vistooma needs to be able to dynamically calculate the set algebraic and mathematical descriptions of the areas of the screen which you pass the cursor over, such as in the following picture:



Once a Venn diagram has been created, Vistooma needs to be able to distinguish the various parts of the diagram such as, in the case of a Venn diagram with 2 sets: A, B, $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$, $A \triangle B$, and all possible combinations of them, as well as the complements of these parts and all possible combinations of those, such as A^C, B^C, $(A \cap B)^C$, $(A \cup B)^C \cap B^C$, $(A \setminus B)^C \cup B^C$, $(B \setminus A)^C$, etc.

As sets can be constructed from many parts as in the example below,

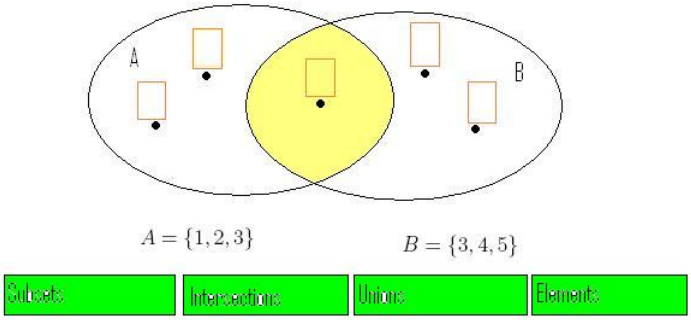


Vistooma needs to allow for selecting sets built up by various parts of the Venn diagram and identify them as a whole, and still apply the algorithms to describe them in the 2 ways that haven't been employed in creating them.

1.1.2 Software Requirements for Working with Elements and Subsets

This option requires the same functionality as in the above section, but in addition it requires a routine to read a short, typed-in list of elements, and somewhat randomly generate a small number of related sets in such a manner that some of them are subsets with or without repeated elements (as it can be a challenge for students to understand that $\{1, 2, 3\}$ and $\{1, 1, 2, 2, 3, 3, \}$ denote the same set), some of them have non-trivial intersection with the original set, and some of them have an empty intersection with the original set.

Vistooma will then generate a Venn diagram with blanks and dots, where you can type in the correct elements, as below:

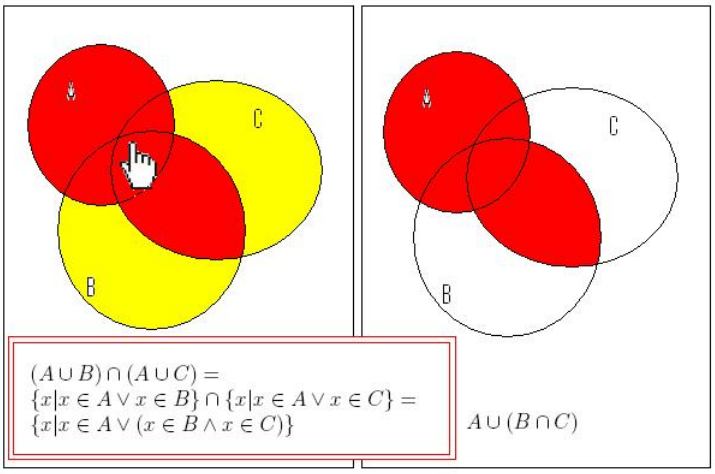


Vistooma will return an error message if you type in an incorrect element, such as typing 4 into the space in $A \cap B$ in the above picture.

Clicking on one the four green buttons at the bottom of the page will generate a true or false question, such as "$\{3,4\} \subseteq A$?" (for Subsets), "$A \cap B = \{5\}$?" (for Intersections), "$A \cup B = \{1,2,3,4,5\}$?" (for Unions) and "$2 \in A$?" (for Elements).

1.1.3 Software Requirements for Proving Set Theoretic Identities

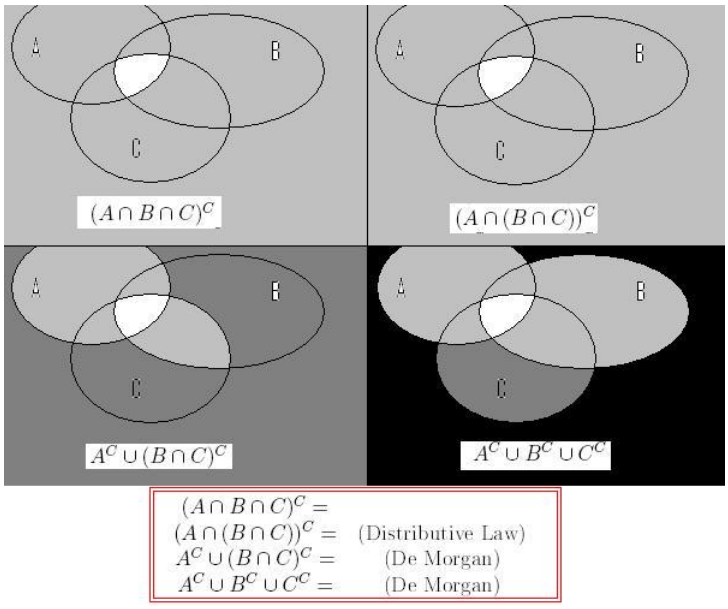
Once the functionalities of the basics are in place, proving set theoretic identities just requires that Vistooma can split the screen and allow the same functionality in each of the subdivided windows. However, as space might become a problem, I suggest that the full 3 way descriptions only pop up in a dialogue box if you click on the selected area, whereas when you don't click, you just get the Venn diagram and the set algebraic description as in the picture below:



1.1.4 Software Requirements for Doing Set Algebra

Set algebra is done by manipulating set algebraic expressions using a clearly defined set of rules. The challenge is that the set algebraic expressions to be manipulated might consist of composite expressions themselves, such as $(A \cup B) \cap (A \cup C)$, in which case the expressions $(A \cup B)$ and $(A \cup C)$ play the roles of both sets and expressions to be evaluated. Vistooma needs to parse a set algebraic expression in a way so that it can deal with parentheses as atoms and as expressions.

In order to facilitate the set algebra using Vistooma, the student will be provided with a command line in which to type in the next step of the manipulations, and the screen will dynamically be split each time you type in a new line and the corresponding Venn diagrams will be generated to allow you to see if you're still dealing with the same set. In the bottom of the screen, Vistooma will keep a list of your manipulations as well as the rule you used, such as in the following picture:



In case the student types in a wrong expression, Vistooma will return an error message and draw the Venn diagram to show the difference.

Duality is a part of set algebra that arises from swapping certain symbols with each other, which is a very simple algorithm, and otherwise use the same rules for set algebraic manipulations as above. Vistooma needs to be able to dualise, and again to parse in such a way that parentheses are treated as both atoms and expressions (when you dualise, you dualise the parentheses first, and then the outer expression).

1.1.5 Software Requirements for Worked Examples

This requires a simple database of worked examples of the various types, along with a random generator which will allow the student to generate and read through an example of the type that she needs. The various areas are: "Elements and Subsets", "Prooving Set Theoretic Identities", "Doing Set Algebra Without Duality", and "Duality".

1.1.6 Software Requirements for the Menus

The drop down menu "Sets" provide the following possibilities: "Create Set", which will generate a set that you can drag and drop to any part of the screen that you want. If you create more than one set, Vistooma will dynamically name them, beginning with A.

You may also choose to "Define Set Using Set Algebraic Notation", which provides you with a command line and a charmap table from which you can select the set algebraic symbols you need ($\cap$, $\cup$, $\setminus$, set^C), while you type in letters and parentheses form the keyboard.

If you choose "Define Set Using Mathematical Notation", you will get a few more symbols to choose from, i.e. $\{$, $\}$, $\in$, $\notin$, $|$, and you type in your description just as above.

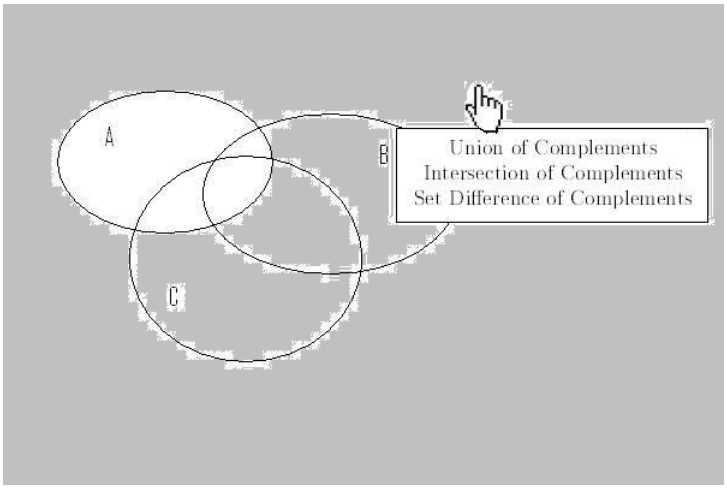
Rightclicking on the letter denoting any set lets you rename it.

Clicking on any part of the Venn diagram depicting sets will provide you with a menu that lets you "Select", "Exclude" or "Ignore" that part of the Venn diagram. It has to identify which part of the diagram you're looking at, e.g. if you click on $(A \cap B \cap C)$, you'll be able to choose between selecting $(A \cap B \cap C)$, excluding $(A \cap B \cap C)$ or ignoring $(A \cap B \cap C)$ (the latter in case you clicked by mistake), or selecting the complement of $(A \cap B \cap C)$.

Clicking once on the Venn diagram outside of the sets will provide you with a menu to allow you to choose a set complement, and clicking again will allow you to do set algebra with the set complements, choosing between intersection of set complements, union of set complements and set difference of set complements, each of which providing you with a sub-menu of the possible combinations.

That is, if you have a Venn diagram of 3 sets, A, B and C, then clicking once outside of the sets lets you choose between A^C, B^C and C^C. If you choose A^C, then clicking again lets you choose between "Union of Set Complements", "Intersection of Set Complements" and "Set difference of Set

Complements".



"Union of Set Complements" has a sub-menu which lets you choose between $A^C \cup B^C$ and $A^C \cup C^C$ (recall that A^C has already been selected), whereas "Intersection of Set Complements" has a sub-menu that lets you choose between $A^C \cap B^C$ and $A^C \cap C^C$, and finally "Set Difference of Set Complements" lets you choose between $A^C \setminus B^C$ and $A^C \setminus C^C$.

In the case of more (or fewer) sets, Vistooma calculates the possible combinations and generates the sub-menus accordingly.

The menu items "View" lets you change the way things are displayed (not necessary in the beta-version, I guess), and the menu item "Tools" lets split the screen, either in 2 to make a set theoretic identity proof or dynamically if you wish to do set algebra, or select and expression to dualise.

1.1.7 Software Requirements for the Optional Settings

Optional settings sort under View. The ones I can imagine are to represent the sets by different shapes, disable to simultaneous 3 way descriptions, stop shading sets that are overlapping more than once in darker colours (I find it instructional, but others may not agree), or choosing which colours you want (the default is shades of red, including yellow and orange, for sets, and shades of grey for set complements).

1.2 "User Manual": Elements of Sets, Subsets

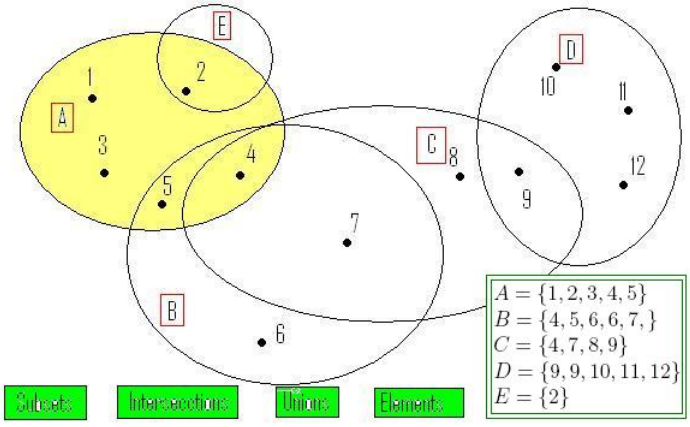
One of the axioms of set theory is that "a set is uniquely defined by its elements". In the set algebraic notation, we do not concern ourselves with the elements, but in the mathematical descriptions, we do describe the set

by giving a way of identifying its elements.

Under the menu item "Sets", you have the option "Elements and Subsets". If you choose this menu item, Vistooma will give you a dialogue box with a charmap table in which to type in a finite set defined by it's elements, e.g. $\{1, 2, 3, 4, 5\}$. This set will automatically be denoted by A.

After you've finished typing in your set, Vistooma will somewhat randomly generate some other sets, denoted by B, C, D etc., such that some of them are made out of the elements of A, some of them are made out of the elements of A with repetitions, some of them contain some elements of A and some elements which are not in A, and some of them have an empty intersection with A.

Vistooma will then calculate how many elements the various sets have in common, and generate a Venn diagram with the right amount of spaces denoted by dots, allowing you to type in the elements in the appropriate places, ending up with a picture such as the following:



If you choose "Subsets", Vistooma will ask a question such as "$A \subseteq E$?", and you have to choose between "True" or "False".

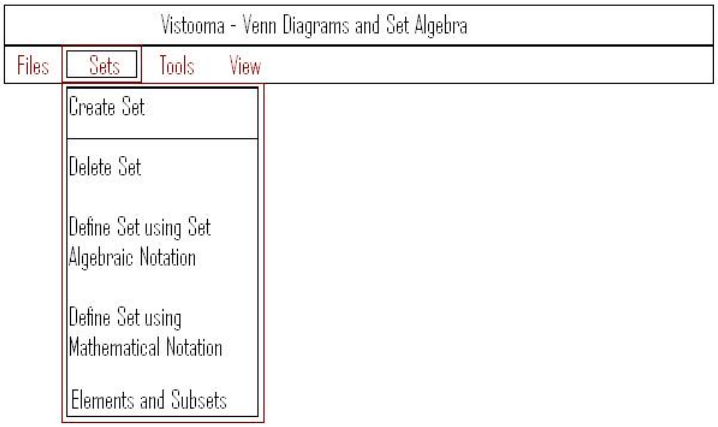
If You choose "Intersections", Vistooma will ask a question such as "$A \cap D = \emptyset$?", and you have to choose between "True" or "False".

If You choose "Unions", Vistooma will ask a question such as "$A \cup D = \{2\}$?", and you have to choose between "True" or "False".

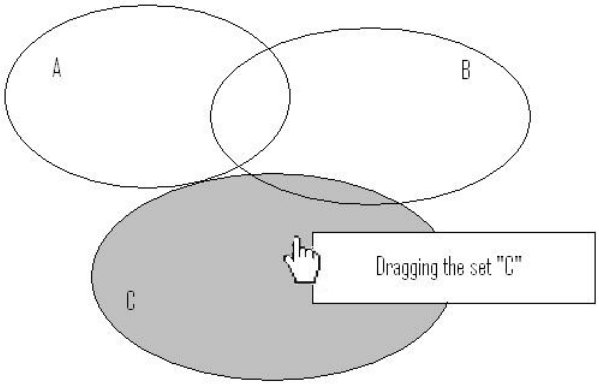
Finally, if you choose "Elements", Vistooma will ask a question such as "$12 \in A$?", and you have to choose between "True" or "False".

1.3 "User Manual": The Basics

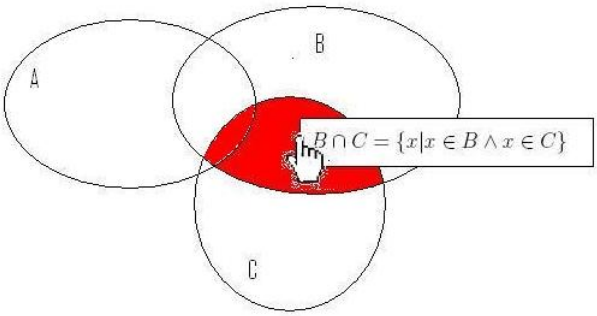
To begin, create as many sets as needed by choosing "Create Set" from the drop down menu "Sets". Vistooma will dynamically assign letters to the sets created, starting with A. It is possible to change the names of the sets by right clicking on the set and choosing "Rename Set". The other menu items under "Sets" lets you switch between "click to mark functionality" and a dialog box which lets you either enter the sets you want to see using set algebraic notation from a charmap table (such as "$A \cap C$"), or enter them by using a mathematical description (such as "$\{x|x \in A \wedge x \in C$"), also choosing the symbols from the charmap table.



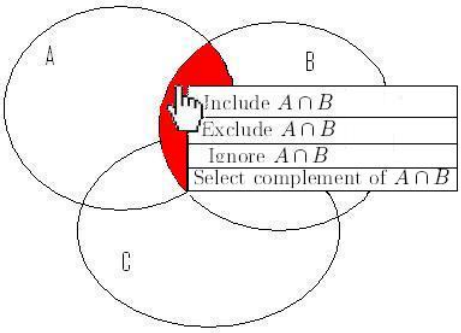
Drag and drop the sets to create a Venn diagram that's suitable for your purposes:



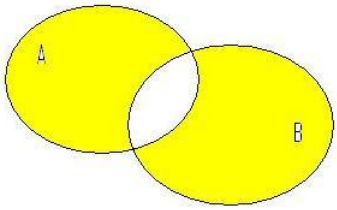
If you let the cursor pass any part of the Venn diagram once you've created it, Vistooma will tell you what it is:



Now, you can select parts of the sets by clicking on them and choosing what to do:



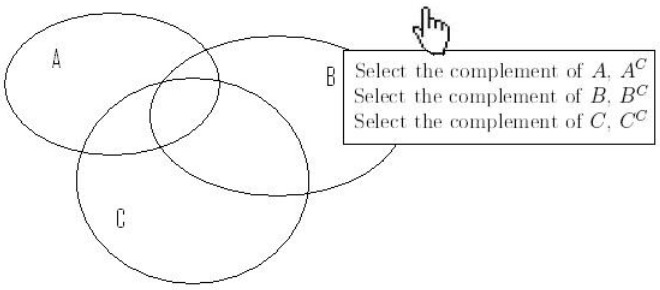
The default of Vistooma is to give you a full (and dynamically updated) description of the set you have selected in order to make you appreciate the different ways of writing it, as in the example below. You can remove the tick from this setting in "View".



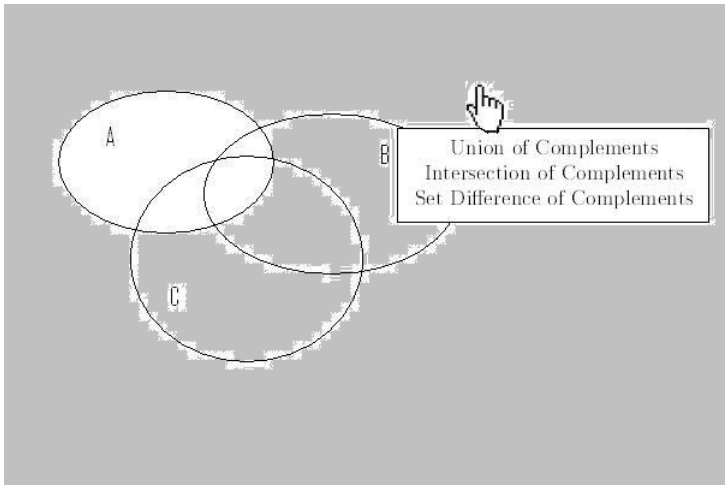
$$A \triangle B = (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B) =$$
$$\{x | x \in A \wedge x \notin B\} \cup \{y | y \in B \wedge y \notin A\} =$$
$$\{x | x \in A \vee x \in B \wedge x \notin \{y | y \in A \wedge y \in B\}\}$$

Showing the Symmetric Difference between the sets A and B

If you click outside of the sets, Vistooma will let you work with set complements:



Once you have selected one complement, you can click again to select more complements and perform set algebraic operations with them. The 3 items in the drop down menu lets you choose submenus, e.g. "Union of Complements" will let you choose between "$A^C \cup B^C$", "$A^C \cup C^C$" and "$B^C \cup C^C$" in a Venn diagram such as this with 3 sets.

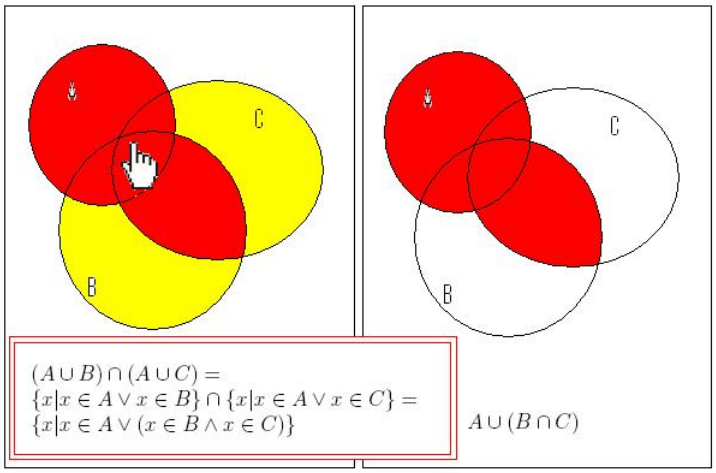


The menu item "View" lets you switch between various modes of drawing the diagrams, e.g. using square sets instead of ellipsical ones or not colouring sets that are overlapping more than one other sets in a darker colour, and so on.

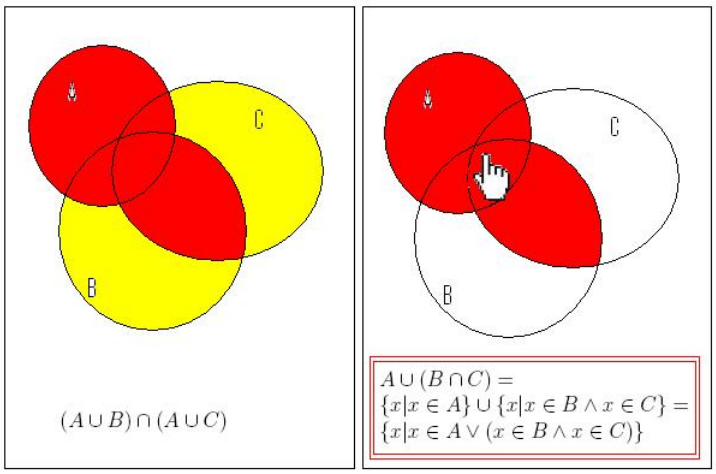
The menu item "Tools" lets split the screen in order to prove set theoretic identities.

1.4 "User Manual": Using Vistooma to Prove Set Theoretic Identities

Once you've split the screen and marked the areas of the sets that you want to work with, double clicking on a marked area will give you a set algebraic and mathematical description of it:

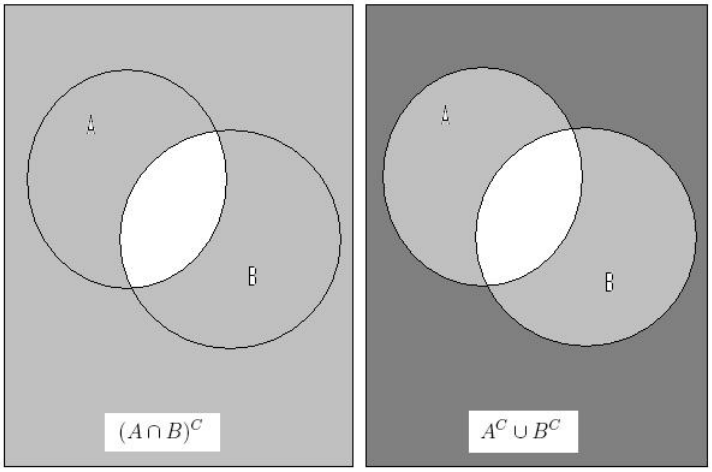


If you need to compare, click on the other side of the diagram:



Showing the identity $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$

Another example:



Showing De Morgan's rule $(A \cap B)^C = A^C \cup B^C$

1.5 "User Manual": Using Vistooma to do Set Algebra

An important part of set theory is the set algebra, which is the rules for manipulating with sets in order to generate new sets or finding other - usually simpler - ways of writing the sets we consider.

1.5.1 Set Algebra

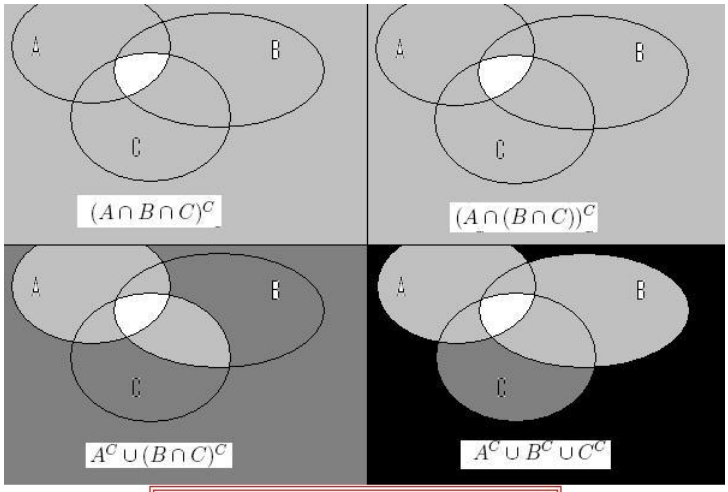
In the Set Algebra, the manipulations we do use the following set of identities or rules, if you will. Note that "U" denotes the Universal Set, which contains everything in the particular environment that you're looking at. It is normally omitted in the set descriptions.

$A \cup A = A$ and $A \cap A = A$	Idempotent Laws
$A \cup B = B \cup A$ and $A \cap B = B \cap A$	Commutative Laws
$A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive Law
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive Law
$A \cup \emptyset = A$	Identity Law
$A \cap U = A$	Identity Law
$A \cup U = U$	Identity Law
$A \cap \emptyset = \emptyset$	Identity Law
$(A^C)^C = A$	Involution Law
$A \cup A^C = U$	Complement Law
$A \cap A^C = \emptyset$	Complement Law
$U^C = \emptyset$	Complement Law
$\emptyset^C = U$	Complement Law
$(A \cup B)^C = A^C \cap B^C$	De Morgan's Rule
$(A \cup B)^C = A^C \cap B^C$	De Morgan's Rule

If you type in any set theoretic expression such as

$(A \cap B \cap C)^C$

Vistooma will evaluate it and draw the appropriate Venn diagram. Step by step, it will ask you to type in the next expression and it will then draw the Venn diagrams to let you check if your new expression leads to the same set as the original one, dynamically splitting the screen and building a table of the identities used. If you type in a wrong expression, it will return an error message and let you retry until you get it right. In this case Vistooma will end up with a picture such as the following:



$(A \cap B \cap C)^C =$
$(A \cap (B \cap C))^C =$ (Distributive Law)
$A^C \cup (B \cap C)^C =$ (De Morgan)
$A^C \cup B^C \cup C^C =$ (De Morgan)

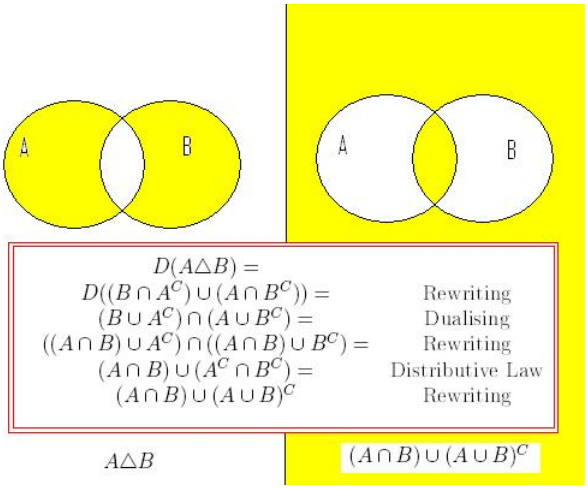
By default, Vistooma is set up to show the manipulations you make in a dialogue box, indicating the rule used after each manipulation. You can remove this functionality by un-ticking the option in "View".

1.5.2 Duality

Another part of set albegra is duality, which allows us to find new identities simply by taking the dual of an already known identity, using the following rules as well as the set algebraic rules above:

$\cup$ is replaced by $\cap$
$\cap$ is replaced by $\cup$
U is replaced by $\emptyset$
$\emptyset$ is replaced by U

When you've typed in a set theoretic expression, or created a set, you can select "Dualise" from "Tools". Vistooma will dynamically split the screen and allow you to create the dualised set in the same manner it lets you do set algebra, except in the end you'll be shown the original set and the dualised set for comparison. In dualising, you do not end up with the same set as you started with, so you do not need to ensure that the expressions you're working with are all describing the same set. Vistooma will generate an error message if you make a set theoretic calculation error.



1.6 "User Manual": Worked Examples

Vistooma provides a selection of worked examples for each module. Here, you can randomly generate a worked example to get a feel for the functionalities or to practise your understanding by looking at examples. In Module 1, you can choose between randomly generating examples of Proving Set Theoretic identities, Doing Set Algebra without Duality and Duality.