VISTOOMA, VIsualisation TOOl for MAth. Module 2: Venn Diagrams, Logical Argumentation and Counting Principles

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Contents

1 Module 2: Venn Diagrams, Logical Argumentation and Counting Principles 1

1.1 Software Requirements for Module 2: Venn Diagrams, Logical Argumentation and Counting Principles . . . 2

1.1.1 Software Requirements for Elements of Sets, Subsets . 2

1.1.2 Software Requirements for Venn Diagrams and Logical Argumentation . . . 2

1.1.3 Software Requirements for The Inclusion-Exclusion Counting Principle . . . 3

1.1.4 Software requirements for Menus . . . 4

1.1.5 Software Requirements for Worked Examples . . . 4

1.2 "User Manual": Elements of Sets, Subsets . . . 4

1.3 "User Manual": Venn Diagrams and Logical Argumentation . 5

1.4 "User Manual": The Inclusion-Exclusion Counting Principle . 6

1.5 "User Manual": Worked Examples . . . 8

1 Module 2: Venn Diagrams, Logical Argumentation and Counting Principles

It is a common practise to use Venn diagrams to sustain logical argumentation, at least in basic abstract math. Lewis Caroll, the author of "Alice in Wonderland", was very fond of this exercise, although he called it the Logic Game. It consists of being given a number of statements such as:

My saucepans are the only things I have that are made of tin.
I find all Sophie's presents very useful.
None of my saucepans are of the slightest use.

identifying each statement with a set and drawing a suitable Venn diagram, which in this case would tell us that Sophie has not given me anything that was made out of tin, nor any saucepans (assuming that a thing is either useful or useless and that no third option exists).

A related exercise is the counting of elements of a union of sets with non-empty intersections, which is why it figures in this module too.

This module relies to a large extend of the functionalities of Module 1.

1.1 Software Requirements for Module 2: Venn Diagrams, Logical Argumentation and Counting Principles

This Module recycles most of the functionalities of Module 1, with just a few additions.

1.1.1 Software Requirements for Elements of Sets, Subsets

See description in Module 1: Venn Diagrams and Set Algebra. This is a recycling of that part of Vistooma, but since the idea is that you may just download the modules that you need, it needs to figure here too.

1.1.2 Software Requirements for Venn Diagrams and Logical Argumentation

Apart from the software needed to generate Venn diagrams from descritions of set theoretic relations described in Module 1, Vistooma needs a "Turn your everyday language into set theoretic language Wizard", taking the students through the steps outlined below, and eventually generating a Venn diagram from the set algebraic expressions the student will create using the wizard:

Step	Instruction	Examples
1) Consider the first statement given	Identify the universal set	"All my old shoes are dirty" tells us that the universal set is that of (my) shoes
2) Read the statement carefully	Identify any sets mentioned	"All my old shoes are dirty" mentions 2 sets, {Old Shoes} and {Dirty Shoes}, and "I keep all my modern shoes clean" also mentions 2 sets, {Clean Shoes} and {Modern Shoes}
3) Consider your sets	Identify the set theoretic relations between them	"All my old shoes are dirty" tells us that {Old Shoes} $\subseteq$ {Dirty Shoes}, and "I keep all my modern shoes clean" tells us that {Modern Shoes} $\subseteq$ {Clean Shoes} and that {Dirty Shoes} $\cap$ {Modern Shoes} $= \emptyset$
4) Return to step 2 with the next statement	Note that it might be about sets already defined	"All shoes that Sophie gave me are in fashion" adds another set, {Shoes that Sophie's gave me}, but it also mentions the already identified set {Modern Shoes}, as modern shoes are in fashion
5) Move on to step 3 with all the sets identified	Don't forget your sets from the first statements	From "All shoes that Sophie gave me are in fashion", we can add that {Shoes that Sophie's gave me} $\subseteq$ {Modern Shoes}
6) Repeat steps 2 through 5 until you have looked at all the statements given	Type the sets and the relations between them into the dialogue box	Make your conclusions from the Venn diagram Vistooma generates

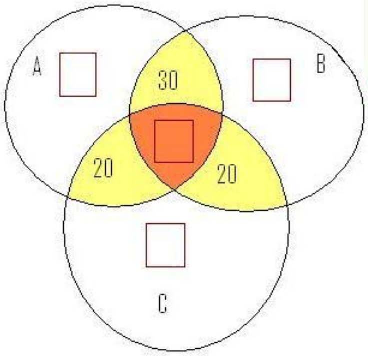
1.1.3 Software Requirements for The Inclusion-Exclusion Counting Principle

Apart from the routines needed to draw Venn diagrams from set theoretic relations outlined in Module 1, Vistooma needs a functionality to let you define the number of elements in a set and the number of elements in an intersection, preferably by using a wizard to let you define it.

After getting the appropriate input, Vistooma needs count the number of sets and then apply the inclusion-exclusion formula with that number of sets to calculate the unknown numbers of elements, and then generate a Venn diagram similar to that of Elements of Sets, but this time the spaces are for the amount of elements in that part of the diagram such as below:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C)$$
$$-n(B \cap C) + n(A \cap B \cap C)$$

$A = \{\text{Brown Kitenge}\}$
$B = \{\text{Blue Kitenge}\}$
$C = \{\text{Green Kitenge}\}$



$n(\{\text{Brown Kitenge}\}) = 80$
$n(\{\text{Blue Kitenge}\}) = 50$
$n(\{\text{Green Kitenge}\}) = 30$
$n(\{\text{Brown Kitenge}\} \cap \{\text{Blue Kitenge}\}) = 30$
$n(\{\text{Brown Kitenge}\} \cap \{\text{Green Kitenge}\}) = 20$
$n(\{\text{Blue Kitenge}\} \cap \{\text{Green Kitenge}\}) = 20$
$n(\{\text{Brown Kitenge}\} \cup \{\text{Blue Kitenge}\} \cup \{\text{Green Kitenge}\}) = 100.$

Vistooma will return an error message if the student types in a wrong number of elements, or defines sets that will end up with a negative number of elements in any component of the Venn diagram.

1.1.4 Software requirements for Menus

The menus need to make it possible for the students to choose between "Sets, Subsets and Elements", "Venn Diagrams and Logical Argumentation" and "The Inclusion-Exclusion Counting Principle". Other than this, most of the menu functionality is described in Module 1.

1.1.5 Software Requirements for Worked Examples

This requires a simple database of worked examples of the various types, along with a random generator which will allow the student to generate and read through an example of the type that she needs. The various areas are: "Elements and Subsets", "Venn Diagrams and Logical Argumentation" and "The Inclusion-Exclusion Counting Principle".

1.2 "User Manual": Elements of Sets, Subsets

This functionality is the same as in Module 1. It is needed in Module 2 as well, because we are working with subsets and elements of sets in this module to an even higher extend than in Module 1.

1.3 "User Manual": Venn Diagrams and Logical Argumentation

The way to solve the above puzzle of Sophie's gifts is to carefully read the sentences and parse them in this way: The first statement,

My saucepans are the only things I have that are made of tin.

tells us that the universe we're in (the set U which was generally ignored in module 1, but which is quite important here) is the set of "all my things". Further, it tells us that the set of saucepans is identical to the set of tin objects.

The second statement:

I find all Sophie's presents very useful.

Tells us that we need to more sets, $\{$All my useful things$\}$ and $\{$All Sophie's presents$\}$ and that $\{$All Sophie's presents$\} \subseteq \{$All my useful things$\}$

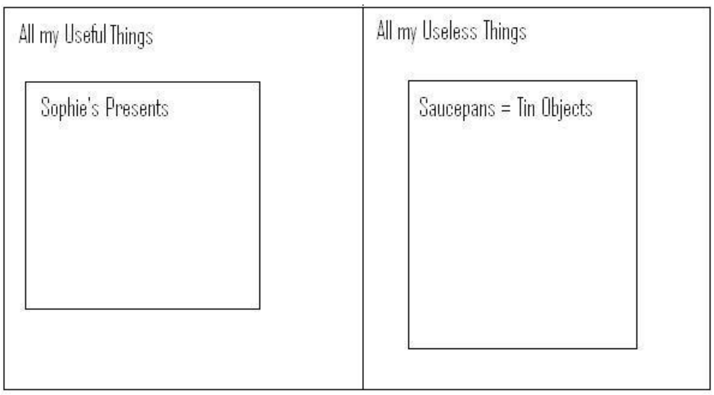
While the last statement:

None of my saucepans are of the slightest use.

tells us that $\{$All my saucepans$\} \cap \{$All my useful things$\} = \emptyset$

while I might have useless things that are not saucepans, or in other words $\{$All my saucepans$\} \subseteq \{$All my useless things$\}$.

Now, we will proceed to draw a Venn diagram of our findings:



As can be seen from the picture, none of Sophie's gifts are saucepans or are made out of tin.

Vistooma facilitates this kind of logical puzzle by taking the statements as inputs, although in a more mathematical form. This is done by guiding you through turning the statements into mathematical expressions step by step, using a Wizard with the steps putlined below:

Step	Instruction	Examples
1) Consider the first statement given	Identify the universal set	"All my old shoes are dirty" tells us that the universal set is that of (my) shoes
2) Read the statement carefully	Identify any sets mentioned	"All my old shoes are dirty" mentions 2 sets, {Old Shoes} and {Dirty Shoes}, and "I keep all my modern shoes clean" also mentions 2 sets, {Clean Shoes} and {Modern Shoes}
3) Consider your sets	Identify the set theoretic relations between them	"All my old shoes are dirty" tells us that {Old Shoes} $\subseteq$ {Dirty Shoes}, and "I keep all my modern shoes clean" tells us that {Modern Shoes} $\subseteq$ {Clean Shoes} and that {Dirty Shoes} $\cap$ {Modern Shoes} $= \emptyset$
4) Return to step 2 with the next statement	Note that it might be about sets already defined	"All shoes that Sophie gave me are in fashion" adds another set, {Shoes that Sophie's gave me}, but it also mentions the already identified set {Modern Shoes}, as modern shoes are in fashion
5) Move on to step 3 with all the sets identified	Don't forget your sets from the first statements	From "All shoes that Sophie gave me are in fashion", we can add that {Shoes that Sophie's gave me} $\subseteq$ {Modern Shoes}
6) Repeat steps 2 through 5 until you have looked at all the statements given	Type the sets and the relations between them into the dialogue box	Make your conclusions from the Venn diagram Vistooma generates

Vistooma will generate the approproate Venn diagram given the set theoretic relations you type in. If you do not type in enough relations to generate a Venn diagram, Vistooma will return an error message.

1.4 "User Manual": The Inclusion-Exclusion Counting Principle

A somewhat related problem is counting the elements of a union of sets with non-trivial intersection. The counting principle of Inclusion-Exclusion tells us how to count the elements of sets with non-trivial intersections using the following formula:

$n(\bigcup_{i=m}^{n} A_i) = \sum_{i=1}^{m} n(A_i) - \sum_{i=2}^{m}\sum_{j=1}^{i} n(A_i \cap A_j) + \sum_{i=3}^{m}\sum_{j=2}^{i}\sum_{k=1}^{j} n(A_i \cap A_j \cap A_k) - \cdots + (-1)^(m-1) n(A_1 \cap A_2 \cap \cdots \cap A_m)$.

This formula allows us to subtract the intersections that we have added one time too much, and then add the intersections that we have subtracted one time too much.

This kind of problems is often posed in a form similar to the following:

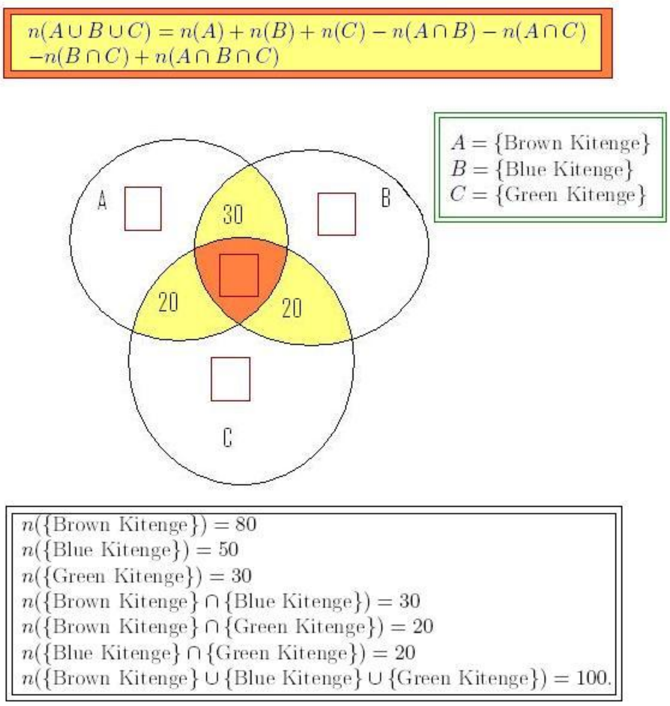
Given 100 pieces of Kitenge:
80 are brown.
30 are green.

50 are blue.
30 are brown and blue.
20 are brown and green.
20 are green and blue.

How many pieces of Kitenge have exactly 3 colours?

First, we identify the 3 sets {Brown Kitenge}, {Blue Kitenge} and {Green Kitenge}, and that we already know that $n(\{\text{Brown Kitenge}\}) = 80$, $n(\{\text{Blue Kitenge}\}) = 50$ and $n(\{\text{Green Kitenge}\}) = 30$, and that $n(\{\text{Brown Kitenge}\} \cap \{\text{Blue Kitenge}\}) = 30$, $n(\{\text{Brown Kitenge}\} \cap \{\text{Green Kitenge}\}) = 20$ and $n(\{\text{Blue Kitenge}\} \cap \{\text{Green Kitenge}\}) = 20$, as well as $n(\{\text{Brown Kitenge}\} \cup \{\text{Blue Kitenge}\} \cup \{\text{Green Kitenge}\}) = 100$.

Given this information by letting you first define the sets and the number of elements, Vistooma generates a Venn diagram such as the following, also calculating the missing numbers but not writing them:



For $m = 3$ we get $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

And since we're interested in $n(A \cap B \cap C)$, we can rewrite the formula to get $n(A \cap B \cap C) = n(A \cup B \cup C) - n(A) - n(B) - n(C) + n(A \cap B) +$

$n(B \cap C) + n(A \cap C) = 100 - 80 - 30 - 50 + 30 + 20 + 20 = 10$, which can be typed into the space in $A \cap B \cap C$. Vistooma will generate an error message if the wrong number is typed in, or if the number of elements in any of the subsections of the Venn diagram will become a negative number.

1.5 "User Manual": Worked Examples

Vistooma provides a selection of worked examples for each module. Here, you can randomly generate a worked example to get a feel for the functionalities or to practise your understanding by looking at examples. In this module, you can find worked examples of using Venn diagrams to argue logically and examples of counting sets with the inclusion-exclusion proncipie.