# VISTOOMA, VIsualisation TOOl for MAth. Module 4: Quantifiers

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## Contents



### 0.1 Software Requirements for Module 4: Quantifiers

#### 0.1.1 Software Requirements for Translating Quantifiers

In order to work with quantifiers, the student must learn how to read various mathematical symbols. In order to facilitate this, Vistooma has a database of sentences such as "All natural numbers $n$ have the property that $n$ is greater than 0" or "There exists an integer $\hat{z}$ such that for all integers $z$, $\frac{\hat{z}}{z}$ is equal to $\hat{z}$" or "Every integer $z$ different from 0 has the property that it is either greater than 0 or smaller than 0", combined with their symbolic form.

Vistooma will randomly generate an everyday language statement, and the student is presented with a charmap table and a dialogue box in which to type in the symbolic representation of that statement, such as below:

| | |
|---|---|
| Statement: | "Every integer $z$ different from 0 has the property that it is either greater than 0 or smaller than 0" |
| Type in the symbolic representation: | |

In which the student can type in his translation of the statement. If the translation is wrong, Vistooma will return an error message such as "Please try again. Your symbolic expression doesn't mean the same as the statement", and if it is correct, the output will be along the lines of "That is correct. "Every integer $z$ different from 0 has the property that it is either greater than 0 or smaller than 0" translates into "$\forall z \in \mathbb{Z} \setminus \{0\} : z > 0 \vee z < 0$"."

In order to achieve this, Vistooma must have a database of such statements and a random generator which will generate a dialogue box such as the above as well as a routine to compare the typed-in expression with the correct translation. Note that $\forall z \in \mathbb{Z} \setminus \{0\} = \forall z \in \mathbb{Z} : z \neq 0$, and that $a \vee b = b \vee a$ and $a \wedge b = b \wedge a$, so both versions must be accepted as a correct answer.

### 0.1.2 Software Requirements for Negating Quantifiers

Negating quantifiers is done by following a very simple algorithm. A statement such as $\exists n \in \mathbb{N} : \forall m \in \mathbb{Z} : n + m \neq 0$ is negated by parsing the statement delimited by the colons as folows $(\exists n \in \mathbb{N}) : (\forall m \in \mathbb{Z}) : (n + m \neq 0)$, and sequencially negating each section of the statement except from the last by changing the following symbols the following way:

| Symbol | Replaced by |
|---|---|
| $\forall$ | $\exists$ |
| $\exists$ | $\forall$ |

The last part of the statement is negated by changing the following symbols the following way:

| Symbol | Replaced by |
|---|---|
| $\in$ | $\notin$ |
| $\notin$ | $\in$ |
| $=$ | $\neq$ |
| $\neq$ | $=$ |
| $<$ | $\geq$ |
| $\leq$ | $>$ |
| $>$ | $\leq$ |
| $\geq$ | $<$ |

The student will be presented with a dialogue box in which to type in the statement using a charmap table, and then the negation when prompted, and Vistooma will return an error message if the negation is wrong. So the routines needed, apart from routines to read the mathematical symbols, are routines to build a dialogue box in which to type in a statement, a negating function and a routine to compare the student's negation with the correct one, telling if it is correct or where it went wrong (See "User Manual").

### 0.1.3 Software requirements for the Menus

The Menus will need to allow the students to choose between translating expressions with quantifiers and negating expressions with quantifiers.

### 0.1.4 Software Requirements for Worked Examples

The database of worked examples need to contain examples of translations between everyday language and symbolic language, and examples of negations.

## 0.2 "User Manual": Translating Quantifiers

For you to learn how to read expressions with quantifiers, Vistooma contains a database of everyday language statements. When you choose to translate quantifiers, Vistooma randomly generates an everyday language statement in a dialogue box with a blank for you to type in the translation such as below:

| | |
|---|---|
| Statement: | "For every natural number $n$, there exists an integer $m$ such that the sum of $n$ and $m$ is 0" |
| Type in the symbolic representation: | |

You will be provided with a chapmap table such that you can choose the characters which do not figure on a keyboard.

| | |
|---|---|
| Statement: | "For every natural number $n$, there exists an integer $m$ such that the sum of $n$ and $m$ is 0" |
| Type in the symbolic representation: | $\forall n \in \mathbb{N} : \exists m \in \mathbb{Z} : m + n = 0$ |
| **That is correct!** | "For every natural number $n$, there exists an integer $m$ such that the sum of $n$ ans $m$ is 0" translates into $\forall n \in \mathbb{N} : \exists m \in \mathbb{Z} : m + n = 0$ |

If you type in a wrong translation, Vistooma will generate an error message.

| | |
|---|---|
| Statement: | "For every natural number $n$, there exists an integer $m$ such that the sum of $n$ and $m$ is 0" |
| Type in the symbolic representation: | $\forall n \in \mathbb{N} : \exists m \in \mathbb{Z} : m + n = 0$ |
| **Error!** | "For every natural number $n$, there exists an integer $m$ such that the sum of $n$ and $m$ is 0" does not translate into $\forall n \in \mathbb{N} : \exists m \in \mathbb{Z} : m + n \neq 0$ |

## 0.3 "User Manual": Negating Quantifiers

If you choose to negate quantifiers, Vistooma will provide you with a dialogue box and a charmap table in which to type in your expression to negate such as below:

| | |
|---|---|
| Statement: | |
| Type in the negation: | |

Once you type in your statement to negate, e.g. $\exists n \in N : n \leq 0$,

| | |
|---|---|
| Statement: | $\exists n \in N : n \leq 0$ |
| Type in the negation: | |

Vistooma automatically negates it and compares your negation with the correct one, telling you if you made a mistake:

| | |
|---|---|
| Statement: | $\exists n \in N : n \leq 0$ |
| Type in the negation: | $\forall n \in \mathbb{N} : n \geq 0$ |
| **Error:** | The negation of $\leq$ is not $\geq$ |

Or if your translation is correct:

| | |
|---|---|
| Statement: | $\exists n \in N : n \leq 0$ |
| Type in the negation: | $\exists n \in \mathbb{N} : n \geq 0$ |
| **That is correct!** | The negation of $\exists n \in N : n \leq 0$ is $\forall n \in N : n > 0$ |

## 0.4 "User Manual": Worked Examples

Vistooma provides a selection of worked examples for each module. Here, you can randomly generate a worked example of a translatipon between everyday language and symbolic representations or of negations of symbolic expressions to get a feel for the functionalities or to practise your understanding by looking at examples.