VISTOOMA, VIsualisation TOOl for MAth.
Module 5: Truth Tables

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Contents

1 Module 5: Truth Tables

The first version of mathematical logic that the student is presented with is that of truth tables. As with most mathematical logic, the truth tables are only conerned with the *shape* of arguments and do not concern the *content* of the arguments, such as the truth table for $p \rightarrow q$ shown below.

p	q	$p \rightarrow q$
t	t	t
t	f	f
f	t	t
f	f	t

Filling in such a truth table is in part learning a few identities by heart and in part using common sense somewhat in the same way as one solves a puzzle.

The challenge for the student is partly to understand which kind of statements are mathematical propositions and which are not, and partly to learn to regard p and q as placeholders which could mean "Bananas are green before they turn yellow" and "New York is in North America" today, and "$2+2=4$" and "All infinite-dimensional Banach Spaces can be classified by their K-theory" tomorrow.

1.1 Software Requirements for Module 5: Truth Tables

1.1.1 Software Requirements for Mathematical Propositions

A mathematical proposition is a statement that is either true or false. While this concept seems fairly simple, it can be hard to decide whether some statement is a mathmatical proposition or not.

The simplest way of deciding if something is a mathematical proposition is by turning in into a question. If it can be answered by a yes or a no, then it is a mathematical proposition. Vistooma facilitates deciding if some statement which it randomly generates from a predefined list of statements is a mathematical proposition by turning the statement into a question, and asking if how to answer it, see table below:

Statement	Question	Select Answer
"There are 28 girls in Sophie's class"	"Are there 28 girls in Sophie's class"	This question can be answered by a YES This question can be answered by a NO This question can't be answered by YES or NO

i.e. if the statement is "x is an odd integer", Vistooma will turn it into the question "Is x an odd integer?". In this case, the correct answer is "This question can't be answered by YES or NO". When something isn't a proposition as in this case, Vistooma will return an answer such as the following: "The statement "x is an odd integer" is not a mathematical proposition because we can't answer it without knowing what x is". In the above example, the answer will be "The statement "There are 28 girls in Sophie's class" is a mathematical proposition because it is either true or false".

This is somewhat subtle, as a student may very well think that if she doesn't know Sophie, then she can't answer the question as to whteher it is true that there are 28 girls in Sophie's class, whereas she might not think that the statement "x is an odd integer" isn't a proposition as she may not appreciate that she doesn't know the value of x, or because she is confused by the fact that the statement turns into a proposition the moment we assign a value to x. That is, "x is an odd integer" is not a proposition, but "For $x = 4$, x is an odd integer" is a proposition equivalent to "4 is an odd integer".

In order to facilitate this, Vistooma needs a database of statements and whether or not they're propositions, as well as the routines to generate a multiple choice dialogue box such as the above.

1.1.2 Software Requirements for Truth Tables

Once we've learned how to decide whether something is a proposition or not, we can start looking at how to use propositions in mathematical argumentation. Now we forget the content of the propositions and concentrate only on the form in which we put them together.

Truth tables provide a very easy way of confirming the validity of simple composite propositions such as $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$. They do not normally pose much of a challenge for students, except when it comes to ensuring that you have included all possible combinations of the original propositions. Their shortcoming is that they grow exponentially, that is for a composite proposition of n original propositions, we need 2^n entries to explore all possible combinations of the original propositions.

In order to facilitate the students' understanding of truth tables, Vistooma will take an input such as "$p \rightarrow (p \vee q)$", identify that there are the following parts: p, q, $(p \vee q)$ and $p \rightarrow (p \vee q)$ and generate the following truth table:

p	q	$(p \vee q)$	$p \rightarrow (p \vee q)$
t	t		
t	f		
f	t		
f	f		

leaving it up to the student to fill in the blanks and returning an error message if a wrong truth value is typed in.

In order to do this, Vistooma needs routines that can correctly parse a proposition and identify all its parts, generate a truth table and calculate all values so as to return an error message if a wrong truth value is typed in by the student.

1.1.3 Software Requirements for Worked Examples

As the propositions part is basically a database, this database need only contain examples of filled-in truth tables as well as a random generator.

1.2 "User Manual": Mathematical Propositions

In order to assist you understand the concept of a mathematicval proposition, Vistooma has a database of statements, some of which are propositions, and some of which are not. When you choose "Select Statement", Vistooma presents you with a multiple choice dialogue box such as the following:

Statement	Question	Select Answer
"All fish live in the sea"	"Do all fish live in the sea?"	This question can be answered by a YES This question can be answered by a NO This question can't be answered by YES or NO

If you select either "This question can be answered by a YES" or "This question can be answered by a NO", Vistooma will return a message such as the following: "That is correct. The statement "All fish live in the sea" is a mathematical proposition as it is either true or false".

If you select "This question can't be answered by YES or NO", Vistooma will return: "Please try again. Is it true that the question can't be answered by yes or no?".

You may also be presented with something which is not a mathematical proposition:

Statement	Question	Select Answer
"$\frac{x}{3} \in \mathbb{Z}$"	"Is $\frac{x}{3}$ an integer?"	This question can be answered by a YES This question can be answered by a NO This question can't be answered by YES or NO

If you select either "This question can be answered by a YES" or "This question can be answered by a NO", Vistooma will return a message such as the following: "Please try again. Do you know if x is divisible by 3?"

If you select "This question can't be answered by YES or NO", Vistooma will return: "That is correct. The statement "$\frac{x}{3} \in \mathbb{Z}$" is not a mathematical proposition as we do not know the value of x".

1.3 "User Manual": Truth Tables

Vistooma assists you in understanding how to use truth tables in the following way: When you type in a (composite) proposition such as $((p \vee q) \wedge \neg p) \rightarrow q$, Vistooma generates a truth table such as the following:

p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$	$((p \vee q) \wedge \neg p) \rightarrow q$
t	t				
t	f				
f	t				
f	f				

leaving it to you to fill in the empty spaces and returning an error message if you type in a wrong truth value.

1.4 "User Manual": Worked Examples

Vistooma provides a selection of worked examples for each module. Here, you can randomly generate a worked example to get a feel for the functionalities or to practise your understanding by looking at examples of filled in truth tables.