VISTOOMA, VIsualisation TOOl for MAth.
Module 8: Language and Logic

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July 1, 2010

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1 Module 8: Language and Logic

For the student to understand that tautologies really just describe various ways of shaping an argument, and in line with the research that suggests that it is always easier for students to learn math if they can apply it to something a little more concrete, it is useful for them to learn how to translate between a rather abstract symbolic expression such as

$\neg(p \wedge \neg p)$

and an everyday statement such as

It is not possible for the sun to rise and set at the same time.

Here, p could be the proposition "The sun is rising", but could equally well be the proposition "The sun is setting".

Note that "It is not possible for the sun to rise and set at the same time" could equally well be expressed as "The sun never rises and sets at the same time", "The sun can't be rising while it's setting", etc.etc.

Another example of translating symbolic logic into everyday language is applying De Morgans rule $\neg(p \vee q) = \neg p \wedge \neg q$ to change a sentence such as

"It's impossible for rain to fall upwards or for time to go backwards"

into

"It's impossible for rain to fall upwards and it's impossible for time to go backwards."

Appreciating this link between language and logic is one of the more challenging disciplines for the students, and one in which the need to localise the software is even more acute because the subtleties of these expressions are easily lost in a language that is not mastered with fluency.

1.1 Software Requirements for Module 8: Language and Logic

1.1.1 Software Requirements for Mathematical Propositions

This is a recycling of the functionalities from Module 5.

1.1.2 Software Requirements for Language and Logic

There are various ways of appreciating what the symbolic expressions really mean in terms of everyday language.

No.	Task	Problem Example	Solution Example
1.	Connecting a statement with its logical equivalent	"All winds from Kilimanjaro are cold"	"If the wind isn't cold, then it is not from Kilimanjaro" (multiple choice)
2.	Recognising the shape of an argument	"All my cat Francina's kittens are grey, so if a kitten isn't grey, then it isn't Francina's"	$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ (typing in)
3.	Recognising the shape of an argument	"Tanzania is hot and dusty, so for sure it is hot"	$(p \wedge q) \rightarrow p$ (multiple choice)
4.	Applying De Morgan's rules to a statement	"We have mangos and pineapples, but a fruit is never a mango and a pineapple at the same time"	"Either the fruit is not a mango or it is not a pineapple" (multiple choice)
5.	Negating plain language sentences using logical negation	"All the fleas in the carpet have been brought in by my neighbour's brown dog ($\forall$ fleas in the carpet: This flea was bought in by my neighbour's brown dog)"	"There exists a flea in the carpet which hasn't been brought in by my neighbour's brown dog" ($\exists$ flea in the carpet: This flea $\neg$was brought in by my neighbour's brown dog) (sequencial multiple choice)

For task number 1, a simple database which randomly generates a multiple choice dialogue box such as the following is needed:

Consider the statement:	Identify the logical equivalent or equivalents	
	"If the wind is not cold, then it is not from Kilimanjaro"	○
"All winds from Kilimanjaro are cold"	"If the wind is hot, then it is from Arusha"	○
	"If the wind is not from Kilimanjaro, then it is not cold"	○

If the students chooses a wrong equivalent, such as "All winds from Kilimanjaro are cold" ↔ "If the wind is not from Kilimanjaro, then it is not cold", Vistooma returns an error message such as: "Please try again. With $p =$ "The wind is from Kilimanjaro" and $q =$ "The wind is cold", the statement "All winds from Kilimanajro are cold" has the form $p \rightarrow q$. The statement "If the wind is not from Kilimanjaro, then it is not cold" has the form $\neg p \to \neg q$, which is not logically equivalent to $p \to q$." That is, the error messages must be linked with the choices the student makes.

For task number 2, we need a random generator of diaogue boxes such as the following:

Consider the statement	Type in the symbolic equivalent
All my cat Francina's kittens are grey, so if a kitten isn't grey, then it isn't Francina's	

The space to type in the symbolic equivalent should come with a charmap table to allow the student to type in the mathematical symbols. If the students types in a wrong expression such as $(p \to q) \leftrightarrow ((p \wedge \neg q) \to c)$, Vistooma returns an error message such as the following: "Please try again. With $p =$ "The kitten is Francina's" and $q =$ "The kitten is grey", the statement "All my cat Francina's kittens are grey, so if a kitten isn't grey, then it isn't Francina's" is not written symbolically as $(p \to q) \leftrightarrow ((p \wedge \neg q) \to c)$".

For task number 3, we need a dialogue box such as the following:

Consider the statement:	Identify the symbolic equivalent	
	$\neg(p \wedge \neg p)$	○
"Tanzania is hot and dusty, so for sure it is hot"	$(p \wedge q) \to p$	○
	$(p \wedge q) \to (p \vee q)$	○

If the students chooses a wrong expression such as $\neg(p \wedge \neg p)$, Vistooma generates an error message such as "Please try again. In the symbolic expression $\neg(p \wedge \neg p)$, there's only one proposition, p, but in "Tanzania is hot and dusty, so for sure it is hot", we can identify 2 different propositions". If it is the other wrong expression, the error message will read: "Please try again. With $p =$ "Tanzania is hot" and $q =$ "Tanzania is dusty", the symbolic expression $(p \wedge q) \to (p \vee q)$ would translate into "Tanzania is hot and dusty, so it is either hot or dusty", which is not the same as "Tanzania is hot and dusty, so for sure it is hot"." That is, the error messages must be linked to the choices the students makes.

For task number 4, Vistooma needs to generate a multiple choice dialogue box such as the following:

Consider the statement:	Identify the De Morgan equivalent	
	"If the fruit is not a pineapple, then it is a mango"	○
"We have mangos and pineapples, but a fruit is never a mango and a pineapple at the same time"	"Either the fruit is not a pineapple or it is a pineapple"	○
	"Either the fruit is not a pineapple or it is not a mango"	○

If the student chooses one of the wrong expressions, Vistooma generates the following error message: "Please try again. De Morgan's Rules states that $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$. With $p =$ "The fruit is a mango" and $q =$ "The fruit is a pineapple", the sentence you picked is not equivalent to $\neg p \vee \neg q$."

Finally, for task number 5, the sequencial multiple choice (or multiple choice wizard) goes as follows:

Consider the Statement	Identify the parts it consists of	
	$\exists$ a flea in the carpet	○
	$\forall$ the fleas in the carpet	○
"All the fleas in the carpet have been brought in by my neighbour's brown dog"	$\exists$ a flea such that it is in the carpet	○
	the flea was brought in by my neighbour's brown dog	○
	some fleas were brought in by neighbour's brown dog	○

If the student doesn't correctly identify the atomic propositions, Vistooma returns a message such as "Please try again. This sentence starts with a quantification and then assigns the same property to all the items it quantifies." When the student has correctly identified the atomic propositions, it continues like this:

Original Statement: "All the fleas in the carpet have been brought in by my neighbour's brown dog"

Consider the first part of your statement	Identify the negation	
	$\exists$ a flea in the carpet	○
$\forall$ fleas in the carpet	$\forall$ fleas in the carpet	○
	$\exists$ a flea such that it $\notin$ the carpet	○

Negation so far: ...

returning an error message such as "Please try again. "$\exists$ a flea such that it $\notin$ the carpet" is not the correct negation of "$\forall$ fleas in the carpet". ", and then after correct identification of the negation:

Original Statement: "All the fleas in the carpet have been brought in by my neighbour's brown dog"

Consider the second part of your statement	Identify the negation	
	some fleas were brought in by my neighbour's brown dog	○
the flea was brought in by my neighbours brown dog	the flea was not brought in by my neighbour's black dog	○
	the flea was not brought in by my neighbour's brown dog	○

Negation so far: $\exists$ a flea in the carpet ...

returning an error message such as "Please try again. "some fleas were brought in by my neighbour's brown dog" is not the correct negation of "the flea was brought in by my neighbour's brown dog".", and finally:

Original Statement: "All the fleas in the carpet have been brought in by my neighbour's brown dog"

Negation: "There exists a flea in the carpet which was not brought in by my neighbour's brown dog"

In order to achieve this, Vistooma needs a database of sentences broken down to atomic propositions as well as the routines to dynamically build the wizards as the student goes along the process of negating the statement, going on to the next page or returning an appropriate error message in case the student chooses a wrong expression.

1.1.3 Software Requiements for Menus

The menus need to allow the students to choose which kind of matching language with logic they want to perform.

1.1.4 Software Requirements for Worked Examples

Here a single database of worked examples of all 5 kinds and a random generator is needed.

1.2 "User Manual": Mathematical Propositions

See description in Module 5.

1.3 "User Manual": Language and Logic

Vistooma lets you choose between 4 ways of connecting language and logic: From the menu, you can choose between **Sentence to Sentence**, **Shape of Argument (Typing In)**, **Shape of Argument (Multiple Choice)**, **De Morgan** or **Negations**.

Sentence to Sentence gives you a dialogue box such as the following:

Consider the statement:	Identify the logical equivalent or equivalents	
	"If the wind is not cold, then it is not from Kilimanjaro"	○
"All winds from Kilimanjaro are cold"	"If the wind is hot, then it is from Arusha"	○
	"If the wind is not from Kilimanjaro, then it is not cold"	○

It is your task to select the correct equivalent. If you choose a wring one, Vistooma will give you an error message with some pointers to choose the right one.

If you choose **Shape of Argument (Typing In)**, Vistooma generates a table such as the following:

Consider the statement	Type in the symbolic equivalent
All my cat Francina's kittens are grey, so if a kitten isn't grey, then it isn't Francina's	

along with a charmap table to allow you to type in the mathematical symbols. If you type in a wrong expression, Vistooma generates an error message with some pointers to the correct equivalent.

If you choose **Shape of Argument (Multiple Choice)**, Vistooma generates a dialogue box such as the following:

Consider the statement:	Identify the symbolic equivalent	
	$\neg(p \wedge \neg p)$	○
"Tanzania is hot and dusty, so for sure it is hot"	$(p \wedge q) \rightarrow p$	○
	$(p \wedge q) \rightarrow (p \vee q)$	○

And it is your task to select the correct one. If you select a wrong expression, Vistooma generates an error message with some pointers to the correct equivalent.

De Morgan gives you a dialogue box such as the following:

Consider the statement:	Identify the De Morgan equivalent	
	"If the fruit is not a pineapple, then it is a mango"	○
"We have mangos and pineapples, but a fruit is never a mango and a pineapple at the same time"	"Either the fruit is not a pineapple or it is a pineapple"	○
	"Either the fruit is not a pineapple or it is not a mango"	○

Your task is to select the correct equivalent. If you select a wrong expression, Vistooma generates an error message with some pointers to the correct equivalent.

If you choose **Negations**, Vistooma gives you a sentence to negate, guiding you through the process with a negation wizard which step by step allows you to identify the original propositions and quantifications and then negate them one by one, until in the end you can see the original statement as well as the negation:

Original Statement: "All the fleas in the carpet have been brought in by my neighbour's brown dog"

Consider the second part of your statement	Identify the negation	
	some fleas were brought in by my neighbour's brown dog	○
the flea was brought in by my neighbours brown dog	the flea was not brought in by my neighbour's black dog	○
	the flea was not brought in by my neighbour's brown dog	○

Negation so far: ∃ a flea in the carpet ...

1.4 "User Manual": Worked Examples

Vistooma provides a selection of worked examples for each module. Here, you can randomly generate a worked example to get a feel for the functionalities or to practise your understanding by looking at examples. Because there's no real difference between the 5 modes of translating between language and logic, Vistooma just randomly generates one of the 5.